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The Believing Game in Mathematics: Stories in a Discipline of Doubt

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Cover Page Footnote

Shelly Sheats Harkness, Associate Professor, Secondary Education Mathematics, at the University of Cincinnati, was a classroom teacher for 12 years prior to earning her PhD at Indiana University. Her research interests include ethnomathematics, mathematics and social justice, and teachers' "believing" and listening in mathematics classrooms. Sue Mau is Associate Professor in the Department of Mathematical Sciences, Indiana University Purdue University Fort Wayne. Her research interests include prospective teachers' understanding of mathematics and their beliefs about teaching. Catherine Pullin Lane is Mathematics Instructor in the Science, Mathematics and Engineering Division at Clermont College and is a doctoral student in Curriculum and Instruction at the University of Cincinnati. Her research interests include how students develop mathematical thinking skills and how they interact with the concept of proof. Amber Brass is Assistant Director Academic in Middle Childhood Education at the University of Cincinnati. Prior to coming to Cincinnati, she was a middle school mathematics teacher in Iowa and Michigan. Her academic interests include the use of literature in the mathematics classroom and the content and pedagogical knowledge of preservice teachers.

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Introduction

Peter Elbow argues that we can improve our practice of understanding by two conflicting processes: methodological belief and methodological doubt. Both are systematic, disciplined, and conscious efforts. However, believing is an endeavor to find virtues and strengths, no matter how unlikely an idea might seem to the listener or reader, and doubting is an attempt to find flaws or contradictions. The problem, Elbow argues, occurs when methodological doubt “hogs the whole bed.” “Judgment”—making up our mind—should occur only after considering the results of both believing and doubting (*Embracing Contraries* 258): “This [believing] may be our only hope of seeing something faint that is actually

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there which she [the student] is particularly good at seeing but the rest of us are ill suited to see” (259).

In mathematics, doubting appears to dominate when student answers do not match our own. As teachers, we habitually criticize and find flaws with students’ understanding or reasoning, and we take this practice for granted as right and natural. We attach importance to finding errors and misconceptions in students’ computation. When we find only one counterexample, we assume (as logic and the doubting game tell us) that the conjecture cannot be true. In order to play the believing game, we must suspend our own logic, assumptions, and interpretations at least until we attempt first to understand and honor our students’ logic, assumptions, and interpretations. Perhaps because doubting caters to our own understanding or weak understanding of mathematics, it is easier to doubt. When students’ answers or methods do not match our own, it is much harder to attempt to believe.

Most people, as a result of their school mathematics experience, view mathematics as “the discipline of certainty par excellence”; however, mathematicians and mathematics educators have noted that this is not the reality of the nature of mathematics (Borasi 158). Ambiguity, doubt, and uncertainty pervade the discipline but not in the ways in which it is presented in schools and perceived by students. Typically, school mathematics is presented as a set of rules and formulae which must be memorized. Students practice using the rules and formulae to complete worksheet or textbook exercises which are evaluated by the teacher as either right or wrong. However, when mathematics is presented as problems which can be solved logically and creatively, then ambiguity, doubt, and uncertainty may surface. This is because problems can be solved using different methods and can have more than one correct answer depending on the students’ logic, assumptions, and interpretations.

If we conceive mathematical knowledge as certain or absolute, then perhaps it is easier to doubt. If we conceive mathematical knowledge as a cyclical process of “proofs and refutations” which produces increasingly refined results (Lakotos), then believing becomes more plausible. According to Marjorie Siegel and Robert F. Carey’s summary of philosopher C. S. Peirce, “truth” is not the goal of knowledge:

Peirce understands that we have to abandon any hope of knowing something is true once and for all and be satisfied with the idea that we can only be certain about something for the time being. . . . *it is this uncertainty that sets the process of knowledge-making in process.* (21-22, original emphasis)

Non-Euclidean geometries (hyperbolic and elliptic), not widely accepted until the 1900s, called on mathematicians to recognize that mathematics can accommodate logical, yet conflicting, axiomatic systems. The traditional Euclidean geometry which is still the framework for high school geometry today must be taught as only one system of geometry. For example, when students learn that all triangles have angle sums of 180 degrees in their high school geometry classes, this “truth” becomes untrue if they later consider triangles on a spherical surface within the non-Euclidean geometries’ axiomatic system. For example, imagine a triangle connecting the cities of Denver to Cincinnati to Sao Paulo on a flat map and a triangle connecting these cities on a globe. The sums of the angles of these two triangles are not equal.

Offering suggestions which will reform school mathematics, Rafaella Borasi, and additionally Borasi and Siegel, propose the following four pedagogical assumptions:

1. *mathematics* as a *humanistic* discipline in which results are not absolute and immutable but are socially constructed and fallible;
2. *knowledge* as a dynamic process of inquiry, characterized by uncertainty and conflict, which leads to a continuous search for a more refined understanding of the world;
3. *learning* as a generative process of meaning making, enhanced by social interactions;
4. and, *teaching* as providing support for students as they search for their own understanding and as organizing the classroom as a community of learners engaged in creating mathematical knowledge. (2-3, original emphasis)

The practice of methodological belief supports these assumptions. But how might methodological belief play out in everyday mathematics classrooms?

Elbow elucidated strategies for “Learning How to Play the Believing Game” within writing classrooms. To paraphrase:

- * Begin with: The five-minute rule—no criticism is permitted and everyone should try to believe
- * The best place: With a small group of people who trust each other
- * The most natural occasion: During discussions where the issue is in some sense an interpretation
- * How to demonstrate its power: Use it in response to people by simply showing them quietly that you can conditionally believe what they are saying. (*Embracing Contraries* 273-75)

We contend these strategies are certainly possible to adopt in a mathematics class, particularly if we embrace a view of mathematics answers as interpretations and if we do not hold an absolutist view of mathematics. When we view mathematics as a human endeavor, a discipline that is socially constructed and fallible, the believing game becomes more possible.

Our Stories

This section consists of four stories. We open ourselves and the doors to our classrooms in order to show how playing the believing game might play out in the discipline of doubt. According to D. Jean Clandinin and F. Michael Connelly, “Narrative inquiries are shared ways that help readers [teachers] question their own stories, raise their own questions about practice, and see in the narrative accounts stories of their own stories. The intent is to foster reflection, storying, and restorying” (20). We hope to do just that.

Amber reminisces about wishing she had played the believing game when she taught middle school mathematics. Sue tells a story about attempting to un-

derstand the thinking behind wrong answers when she worked with preservice elementary teachers. Catherine's narrative is about attempting to believe her college-level students. And, finally, Shelly tells two stories. In her first story she chronicles the use of believing as a framework for her research, and in her second story she describes her own attempt to believe while observing a student teacher in a high school classroom.

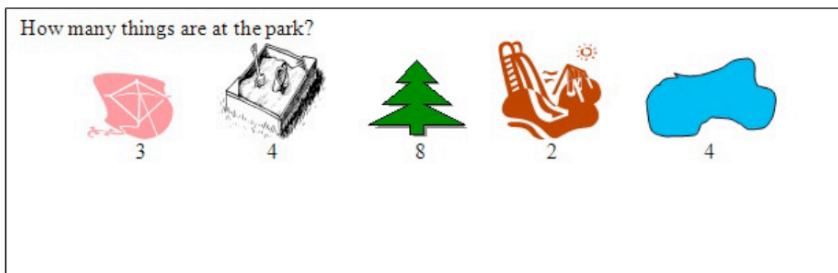
Amber's Story

There were many times, especially in my first years of teaching, when looking back I wish I had used the believing game. Here is an example of one of those times. It shows how the believing game can be used to encourage rich mathematical discourse in the classroom and for teacher reflection.

Mean, median, and mode are concepts that create confusion for students of all ages, both children and adults. Mathematicians consider each of these concepts (mean, median, and mode) to be an average, but the concepts are used in different situations depending on the information being communicated. To help me see my students' understandings of these concepts, I asked my students to create books about mean, median, and mode. Students were asked to define each type of average, highlight important features, and provide an example from everyday life.

In Debbie's description of mode she explained that mode is "the number used the most," and she used this strategy to determine the answer of 4 in her example (see Figure 1). Debbie drew pictures of things she would see at a park and put the number of these items at the park under each picture. Debbie then listed these numbers from smallest to biggest to help her see which number was used the most.

Figure 1: Debbie's Example for Mode



The mathematical definition of mode is the number or item that occurs most frequently in a set of data. I was using this definition, and a doubting lens, as I graded Debbie's example for mode. I am sure that at the time I graded this project I thought, "This is incorrect. The mode of this situation is actually the trees, because there were 8 of them at the park. That is the item that occurs most frequently."

Actually, my thinking was limited and absolute. I needed to remember that numbers and items reside within the definition of mode depending on how the situation is interpreted. Suppose I collected data on how many trees were in five students' yards, and they reported the number of trees in their yards to be 2, 3, 4,

4, and 8. It is most common for there to be 4 trees in a yard (according to this data), and the mode would be numerical, 4. However, I could also collect data on the types of trees in each yard. For example, there could be 2 elm trees, 3 maple trees, 4 apple trees, 4 dogwood trees, and 8 oak trees in a yard. In that case, oak trees occur most frequently, so the mode would be categorical, oak trees.

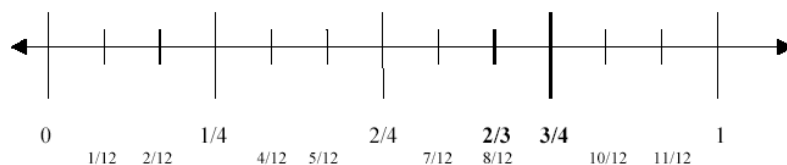
If I had been looking through a believing lens, I would have seen the strength of Debbie's numerical answer of mode since 4 was the number that occurred most frequently. I would have spent more time on this example with my class, and I am left wondering about the rich conversation that could have taken place had we examined Debbie's answer. Perhaps students could have come to a greater understanding of mode and its different appearances, both numerical and categorical. Debbie saw the numerals, and she understood 4 to be the mode since it was the number that was used most frequently which fits the mathematical definition. Looking at the "things in the park" as a list (kite, kite, kite, sandbox, sandbox, sandbox, tree, tree, tree, tree, tree, tree, tree, lake, lake, slide, slide, slide, slide) Debbie might have had a different interpretation of mode, tree, since it is the category that was used most frequently in this example. Perhaps students could have defended the rightness of 4 as the answer or defended the rightness of trees as the answer gaining an appreciation for the ways we can use statistics to show different points of view. Perhaps I could have learned more about the use of mode and more about the thinking of my students had I asked Debbie to share her example with the class.

Sue's Story

Attempting to believe reminded me of a conversation with one of my former colleagues at Purdue Calumet, Erna Yackel. Following a social constructivist perspective (Cobb, Wood, and Yackel), she said that students' answers are typically sensible to them, that students did not purposely give wrong answers or what some might call "stupid answers." Although I doubted her, I became committed to finding out what sense students' answers made to them, on the off chance that she might be right. I became devoted to teasing out their reasoning, even if it was incorrect, or perhaps incomplete, reasoning.

I have been teaching prospective elementary school teachers. Recently some of these students were working on division of fractions. I reminded them that division asks the question "how many of this in that?". Believing that students must make sense for themselves and that a drawing often helps with that process, I asked them to draw a picture for $3/4 \div 2/3$. In other words, they needed to draw a picture that would show how many $2/3$ s are in $3/4$. Before I allowed them to begin their drawings, I asked if the answer should be more than, equal to, or less than one. All agreed that the answer should be greater than one because the divisor ($2/3$) is less than the dividend ($3/4$) (see Figure 2).

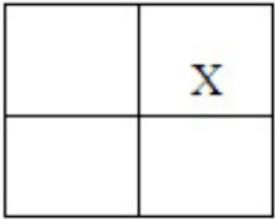
Figure 2: Number Line Illustration of Dividend ($3/4$) and Divisor ($2/3$)



Also before they began working in groups to determine an answer, I made it clear that using the “invert and multiply” rule to get the answer (1 and 1/8) would be insufficient.

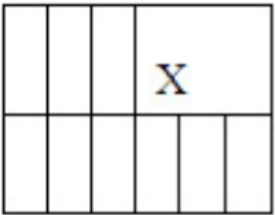
Consequently, when a group gave the answer as 4 and 1/3, I knew that I would have to tease this out carefully. In my best reasoning, I could never have produced that answer, and I certainly could not anticipate how they got it. I asked students to come to the board and “draw their answer.” Since their answer was so different than other answers, they declined my invitation. However, I would not let that rest. How did they begin the problem? How did they arrive at their answer? One of the students from the group sketched a square box, divided it horizontally and vertically into fourths, crossed out one of the fourths and worked with the other three fourths (see Figure 3).

Figure 3: Students Sketch Fourths and Cross Out 1/4.

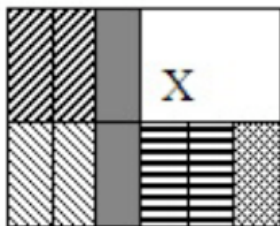


She then divided each of the remaining fourths into thirds, creating 9 twelfths (see Figure 4), although she thought of these twelfths as thirds.

Figure 4: Students Cut Each Fourth Into “Thirds” To Make 9 Twelfths



Then she shaded two of the pieces (thirds in her mind) at a time and counted how many groups of two she had shaded (see Figure 5).

Figure 5: Students Shade “Thirds,” Two At a Time, Counting Up To 4 and $\frac{1}{3}$ 

She could color four groups of two pieces with one piece left over, prompting the answer of 4 and $\frac{1}{3}$.

Their answer was wrong, but their technique had merit, even if they had incomplete understanding. What was less clear to me were the questions I should ask to help them make better sense of this work. How do I help future teachers see the value in wrong answers? How do I help them know why this is numerically wrong and still validate the group's thinking, which has merit? I certainly did not want to leave anyone thinking the answer actually was 4 and $\frac{1}{3}$, but I did want these future teachers to consider believing students' thinking before making pronouncements about right and wrong. All of this happened just minutes before the end of class, making my dilemma seemingly more urgent.

We picked up the discussion again in the next class. The group came to understand how they had misinterpreted the “chunks” of $\frac{2}{3}$ in the problem. The students' solution strategy of splitting the unit, in this case 1, into fourths and then splitting the fourths into thirds caused them to use a divisor smaller than stated in the problem. They created a divisor of $\frac{2}{12}$ (or $\frac{1}{6}$) rather than $\frac{2}{3}$. In the problem, as stated, the divisor was $\frac{2}{3}$, which is the same as $\frac{8}{12}$. Looking at Figure 1 may help the reader literally to see the difference in magnitude and how that affects the numerical solution. When something is cut into one large piece, the answer is less than when that same thing is cut into many smaller pieces. When the students changed the divisor to $\frac{2}{3}$ of $\frac{1}{4}$, they inadvertently changed the size of the divisor to $\frac{2}{12}$ (or $\frac{1}{6}$) which was one-fourth of its original size (see Figures 4 and 5). Consequently when I asked the question “how many of the divisor fit into the dividend?”, they got the wrong numerical answer of 4 and something (the students said $\frac{1}{3}$) rather than 1 and something. Again, their strategy had merit and needed revision.

If this seems confusing, it is. Following the “swirling and colorful incompleteness of [students'] talk” (Ball 733) is never easy, but it is necessary if we believe something is there. As Deborah Loewenberg Ball reminds us, students' thinking and articulation can be clumsy, and “clumsy articulation may not be clumsy or inarticulate at all, but rather it may reflect how the speaker actually understands what he or she is talking about” (735). In this case, the problem with the solution becomes “clumsy” because the unit keeps shifting. In order to understand this solution, the teacher must be able to think from the student's point of view and realize the subtle change in unit size from $\frac{2}{3}$ of one to $\frac{2}{3}$ of one-fourth.

Their strategy, numerically incorrect as it was, allowed us to have a good conversation about the meaning of division and how we compare what is left

over to the divisor in order to get the remainder. Our discussion allowed us to think about the difference in the quality of a strategy and in a numerically correct answer. In effect, my attempts to find the logic in the wrong answer allowed students to rethink their understanding of division of fractions, and it allowed me to rethink how students make sense of division in general. As I have thought about this episode during the last two years, I have also come to realize that I have re-thought my division process. I believe I now have an enhanced relational understanding (Skemp) of the meaning of division. Thanks to the students' thinking and my determination to begin believing, I can better visualize a geometric interpretation of division. What emerging understandings, students' and mine, might be quashed if we do not look at the thinking rather than just the answer?

Catherine's Story

I teach at an open access community college, and most of my students come with a background of struggles with mathematics. This is most definitely the case with the students who take Preparatory Mathematics, a basic review of the fundamental of mathematics. The students who test into this class usually say that they "hate mathematics" and "were never good at it." It was easy to play the doubting game because I could so easily assume that their incorrect answers came from a lack of knowledge.

After reading Shelley Harkness and discussing with her how the believing game could be played out in the mathematics classroom, I decided to take a chance. For the next quarter I was going to try to believe that when the students spoke there would be some kernel of truth in what they said. I had my doubts, but I was intrigued by the idea. I hoped that as a result of my believing the students would become more involved in the class and build some self-confidence. What I didn't expect to happen was that they would show me a new way to think about mathematics.

On the third day of the course, my lesson goal was that students would understand integer (i.e., . . . -3, -2, -1, 0, 1, 2, 3) computation. While I had a few ways of personally understanding this, none felt completely satisfying to me and I did not expect, or I doubted, that my students would be able to create their own understanding of this concept.

We began the class by making sure we all understood why a positive integer times a positive integer is another positive integer. One way to see this is to view multiplication as repeated addition. That is, $3 \times 4 = 4 + 4 + 4 = 12$. The class agreed that repeatedly adding positive integers would result in a positive integer. From there we moved to understanding why a positive integer times a negative integer is a negative integer. The class was quick to accept this makes sense since $3 \times (-4)$ could be written as $(-4) + (-4) + (-4) = (-12)$ and repeatedly adding a negative integer would result in another negative integer.

Then I gave them the challenge: How can we make sense of $(-3) \times (-4)$? After giving them some time to tinker with this idea, I asked for any suggestions. A student in the back of the class offered an explanation, and I prepared myself for trying to believe. The problem was that I could not understand his explanation enough to ask a clarifying question. I knew he said something about subtraction, but that it "didn't work." Determined not to slip into doubting and wanting to understand what this student was trying to say, I asked the class to clarify what

the student had said. To my surprise a student spoke up quickly. In fact, she had the same idea that the other student had. Instead of looking at multiplication strictly as repeated addition, when there are two negatives, they were changing it to repeatedly subtracting a negative integer. They were viewing $(-3) \times (-4)$ as subtracting the integer (-4) three times, that is, they saw it as $(-4) - (-4) - (-4)$. However, because some students had memorized the rules for integer operations (“a negative times a negative equals a positive”) they knew that the answer should be $(+12)$. But calculating the answer to $(-4) - (-4) - (-4)$ one must work left to right: $(-4) - (-4)$ equals (0) and $(0) - (-4)$ equals $(+4)$. After making sure the rest of the class understood what was happening, I asked them to take more time to think about this method and see if the problem could be resolved. Was there a way they could view the product of two negative integers as repeated subtraction of a negative integer?

While I was still pondering this, a lively discussion broke out and the class became convinced they had the answer. Start with zero! That is, $(-3) \times (-4) = 0 - (-4) - (-4) - (-4) = 12$. They then assured me that they could do this with “normal” multiplication, too. $3 \times 4 = 0 + 4 + 4 + 4$. Another student spoke up and said we could view $(-3) \times 4$ as $0 - 4 - 4 - 4 = -12$! They were on to something.

The results from this single day of believing have been far reaching. I now enjoy listening to energetic mathematical conversations and debates on a regular basis. The students are quick to offer suggestions and listen to their peers. And the biggest surprise of all, I learned a new, a more satisfying way of understanding why a negative integer times a negative integer is a positive integer.

Shelly's First Story

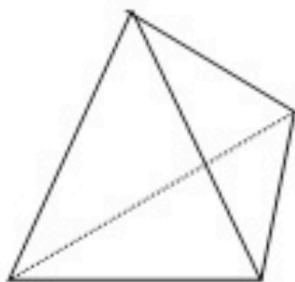
I was a doctoral student, transcribing and analyzing data from videotaped episodes in Sheila's classroom. Sheila (pseudonym) taught a mathematics course, Problem Solving, at a large urban university. My co-researchers and I authored two papers about Sheila's teaching practice. For these papers we used a theoretical framework grounded in motivation goal theory. The students in Problem Solving reported that they were motivated and that Sheila used practices which emphasized learning goals, rather than performance goals (Harkness, D'Ambrosio, and Morrone; Morrone, Harkness, D'Ambrosio, et al.). When students have learning goals, they focus primarily on mastering tasks and learning for learning's sake, they see a direct relationship between effort and learning, and they are willing to put forth more effort to complete challenging tasks (Ames; Dweck; Dweck and Leggett).

Unfortunately, I was not satisfied. I felt that the motivation goal theory framework did not adequately describe Sheila's teaching practice. I wanted to portray the ways that she honored students' mathematical thinking. In a third paper (Harkness), I described one aspect of Sheila's practice: attempting to believe. In fact, although Sheila had no knowledge of methodological belief or methodological doubt, in interviews with me she described her own attempts to believe. One particularly interesting attempt to believe occurred when Sheila was visiting a middle school classroom. Sheila said the teacher asked students for the answer to one of five true-false warm-up exercises: All triangles have three sides. Sheila said,

Kayla raised her hand to say the answer was false, and the teacher responded, “We all know that every triangle has three

sides.” Because the teacher valued my input, I felt comfortable attempting to open the conversation by asking Kayla to tell the class why it was false. “How do you see it? Can you draw it on the board so that we can see what you see?” In response to my questions, Kayla drew a shape which resembled a tetrahedron (see Figure 6).

Figure 6: Tetrahedron



The teacher said, “No. That’s not a triangle. It’s not flat. The answer must be true.” The conversation stopped. However, I wondered if Kayla was looking at, picturing, each side individually, and thinking that there were four sides to this “triangle” and that the sides were flat. Because the teacher made it clear that it was time to move on with the lesson, as the visitor, I kept further questions to myself. I thought about how a rich opportunity to explore three-dimensional space or talk about the geometric language of sides and faces was dismissed by the teacher. (Interview, May 22, 2002)

In her own practice, Sheila looked for sparks of rightness about her students’ mathematics, their solutions, and the thinking behind those solutions. She practiced “unpacking” (Sheila’s language) their mathematical thinking and asking them to clarify their assumptions and interpretations. And, recalling Kayla’s diagram of a “4-sided triangle,” Sheila said that she “rethought” her own mathematics.

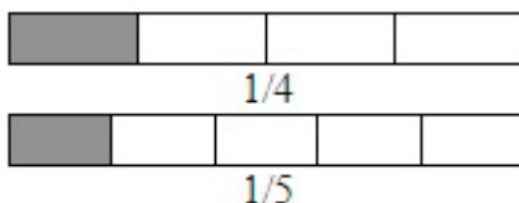
Shelly’s Second Story

During the same period of time that I was writing the third paper about Sheila’s practice in Problem Solving, I visited a mathematics classroom. I was observing a student teacher, and high school students were sitting in pairs playing a game called “Capture.” After each pair of students flipped two playing cards over, the pairs decided which person’s cards named the greatest fraction and that person collected all four cards. For example, if one student flipped over 3 and 4 ($3/4$) and the other flipped over 4 and 5 ($4/5$), the student with $4/5$ took all four cards because $4/5$ is greater than $3/4$. The object of the game was to capture more cards than your opponent. The students were allowed to play the game for about 10 minutes before a large group discussion ensued. As the large group discussion began, the student teacher asked students how they knew which fraction pair was greatest:

Two student pairs shared the two methods that they used: finding common denominators (for example, $3/4 = 15/20$ and $4/5 = 16/20$); using calculators to change the fractions to decimals ($3/4 = 0.75$ and $4/5 = 0.80$). The student teacher acknowledged that both of these were “good” ways to compare the fractions. However, when Sam raised his hand and said that he looked at the “bottom number” and “the one with the lowest bottom number was the greatest,” the student teacher corrected Sam, “Be careful. We know that method will not work.”

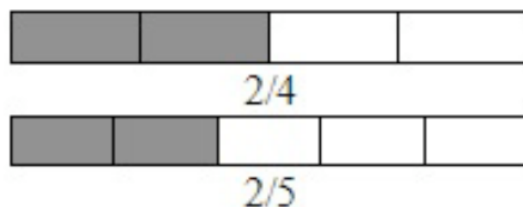
As I sat there, I played the believing game. I thought about unit fractions, fractions with numerators of one, such as $1/4$ and $1/5$ (see Figure 7).

Figure 7: Comparison of Unit Fraction $1/4$ and $1/5$



Sam’s method is always true if he was describing how you know which fraction is largest when both are unit fractions. After thinking about Sam’s method, I later (days later, in fact) realized that his method is also true when fractions with the same area have any common numerators (see Figure 8).

Figure 8: Comparison of Fractions with Common Numerators



However, finding common numerators rather than common denominators is not a customary method for comparing fractions, at least not in most U.S. classrooms. In fact, Sam’s method works if the numerators happen to be the same or if we manipulate fractions to have the same or “common numerators.”

This episode made me wonder. What kind of conversation might have occurred if the student teacher had believed (rather than doubted) and noticed a spark of rightness in Sam’s method? What would have happened if the student teacher had said, “Tell us more. Please give us an example of what you mean.” Would Sam and other students have been more willing to share their thoughts in future conversations? Would Sam have felt empowered? Would the student teacher’s own subject matter knowledge have been impacted?

Elbow says that to learn to play the believing game we should begin with the five-minute rule. No criticism is permitted, and everyone should try to believe. However, for me, the five-minute rule was not the starting point. Both Sheila and I were observers while other teachers taught. We had the opportunity to think deeply about students' understanding because we did not have to "think on our feet." Perhaps learning to play the believing game in everyday mathematics classrooms may be fostered by attempting to believe while observing other teachers, staying out of the conversation, listening to students, and considering how an answer deemed wrong might actually have some kernel of truth. The comfort of observation—watching others play the doubting game while trying to believe—may be how to begin learning to play the believing game in mathematics classrooms.

Conclusion

Mathematics is not "the discipline of certainty par excellence": ambiguity, doubt, and uncertainty filter through. Because of this, we must attempt to make believing a prerequisite for doubting in our mathematics classes. We should endeavor to suspend our own logic, assumptions, and interpretations in order to first try to understand and honor our students' logic, assumptions, and interpretations. When we attempt to believe, we create opportunities for rich conversations about the mathematical content. We also improve the potential for "unpacking" students' logic, assumptions, and interpretations. This "unpacking" may honor students' thinking, and perhaps our students will be motivated to do mathematics. Additionally, we may learn the content that we love in deeper ways.

The aim of interpretive or hermeneutic inquiry is not to write the end of an existing story but to write more hopeful beginnings for new stories (Ellis). We are hopeful that our stories are only the beginnings for us. Perhaps our stories are also beginnings for other mathematics teachers who read them and then envision their own practice as one of attempting to believe in a discipline of doubt. □

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